

## 7-4 Remainder and Factor Theorems

Solve.

$$x^3 + 4x^2 - 15x - 18 = 0$$

- If  $x - 3$  is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -15 & -18 \\ & & 3 & 21 & 18 \\ \hline & 1 & 7 & 6 & 0 \end{array}$$

$x^2 + 7x + 6$

$$(x^2 + 7x + 6)(x - 3) = 0$$

$$(x + 1)(x + 6) \quad x = 3$$

$$x = -1 \quad x = -6$$

$$\{-6, -1, 3\}$$

Solve.

$$x^3 + 7x^2 + 2x - 40 = 0$$

- If  $x - 2$  is a factor.

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 2 & -40 \\ & & 2 & 18 & 40 \\ \hline & 1 & 9 & 20 & 0 \end{array}$$

$0 = (x - 2)(x^2 + 9x + 20)$   
 $(x + 5)(x + 4)$   
 $\{-5, -4, 2\}$

When it is a factor, what can you say about the remainder?

Is it a factor?

$$f(x) = x^3 + x^2 + 3x + 3$$

Is  $x + 2$  a factor?

*No, Remainder*

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 3 & 3 \\ & & -2 & 2 & -10 \\ \hline & 1 & -1 & 5 & -7 \end{array}$$

$$(-2)^3 + (-2)^2 + 3(-2) + 3$$

$$f(x) = x^3 + x^2 + 3x + 3$$

Find  $f(-2)$ .  $= -7$

*Synthetic Subst.*

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 3 & 3 \\ & & -2 & 2 & -10 \\ \hline & 1 & -1 & 5 & -7 \end{array}$$

$$f(x) = x^3 + x^2 + 3x + 3$$

Is  $x + 3$  a factor? *no*

$f(-3) = ?$   
 $-24$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & 3 & 3 \\ & & -3 & 6 & -27 \\ \hline & 1 & -2 & 9 & -24 \end{array}$$

$$f(x) = x^3 + x^2 + 3x + 3$$

Is  $x + 1$  a factor?

yes

$$\begin{array}{r} -1 \overline{) 1 \ 1 \ 3 \ 3} \\ \underline{-1 \ 0 \ -3} \\ 1 \ 0 \ 3 \ 0 \end{array}$$

Find  $k$  such that

$2x^4 + x^3 + 5x^2 - 6x + k \div x + 2$  has a remainder of 5.

$$\begin{array}{r} -2 \overline{) 2 \ 1 \ 5 \ -6 \ k} \\ \underline{-4 \ 6 \ -22 \ 56} \\ 2 \ -3 \ 11 \ -28 \ 56+k \end{array}$$

$$56+k=5$$

$$k=-51$$

$$2x^4 + x^3 + 5x^2 - 6x + k$$

Find  $k$  such that  $x + 2$  is a factor.

$$56+k=0$$

$$k=-56$$

Remainder Theorem (summary)

The remainder of  $f(x) \div (x - a)$  is  $f(a)$ .

Factor Theorem (summary)

The binomial  $(x - a)$  is a factor of  $f(x)$  iff  $f(a) = 0$ . "if + only if"

HW p368 15, 25, 27, 33

Use synthetic substitution to find  $g(3)$  and  $g(-4)$  for each function.

15.  $g(x) = x^3 - 5x + 2$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

25.  $2x^3 - 5x^2 - 28x + 15; x - 5$

27.  $2x^3 + 7x^2 - 53x - 28; 2x + 1$

Find values of  $k$  so that each remainder is 3.

33.  $(x^2 - x + k) \div (x - 1)$